Spacecraft Imaging Observations

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Imaging Observation Modelling

We define the time of an imaging observation as

\[ t = \left( t_{ob} - \frac{1}{2} t_{exp} \right) + \Delta T \]

where \( t_{ob} \) is the end of the exposure in UTC, \( t_{exp} \) is the exposure time, and \( \Delta T \) is the difference between TDB and UTC. The true position of an object (planet, satellite, asteroid, comet) as seen from a spacecraft at time \( t \) (TDB) is

\[ \mathbf{T} = \mathbf{s}(t - \tau) - \mathbf{r}(t) \]

where \( \tau \) is the light travel time from the object to the spacecraft, \( \mathbf{s} \) is object’s position, \( \mathbf{r} \) is the spacecraft position; both positions must be relative to the Solar System barycenter. The computation implicitly involves an iterative procedure to determine \( \tau \) from the relative distance between the object and the spacecraft. The apparent position of the object seen from the spacecraft is found by correcting for stellar aberration (due to the motion of the spacecraft)

\[ \mathbf{A} = \mathbf{T} + \tau \mathbf{v}(t) \]

where \( \mathbf{v} \) is the Solar System barycentric spacecraft velocity. Positions for all spacecraft are available in the form of SPK kernels from NASA’s Navigation and Ancillary Information Facility (Acton, 1996).

For star observations the true position is

\[ \mathbf{T} = \begin{bmatrix} \cos \alpha_\star \cos \delta_\star \\ \sin \alpha_\star \cos \delta_\star \\ \sin \delta_\star \end{bmatrix} \]

where $\alpha, \delta$ are the right ascension and declination of the star, corrected for parallax and proper motion. The apparent stellar position is

$$A = \frac{T + v(t)/c}{|T + v(t)/c|} = T + v(t)/c - (T \cdot v(t)/c) T + O(c^2)$$

where $c$ is the speed of light.

The apparent position is rotated into camera body coordinates:

$$P = R_3 (\Omega) R_1 (-X) R_2 (\Psi) R_3 (\phi) R_2 (90^\circ - \delta) R_3 (\alpha) A$$

where $R_i (\theta)$ is a rotation by angle $\theta$ about axis $i$ and

- $\Omega$ = the camera twist offset angle
- $X$ = the camera cross-elevation offset angle
- $\Psi$ = the camera elevation offset angle
- $\phi$ = the twist angle of the scan platform
- $\delta$ = the declination of the scan platform
- $\alpha$ = the right ascension of the scan platform

Note that the scan platform rotation is an unconventional 3-2-3 rotation. The camera body coordinates are projected into the focal plane via a gnomonic projection

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{f}{P_3} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$

where $f$ is the camera focal length and the $P_i$ are the three components of the vector $P$. The focal plane coordinates are corrected for electromagnetic and optical distortion:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -yr & xr^2 & -yr^3 & xr^4 & xy & x^2 \\ xr & yr^2 & xr^3 & yr^4 & y^2 & xy \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{pmatrix}$$

where $r^2 = x^2 + y^2$ and the $e_i$ are the camera distortion coefficients. The conversion from focal plane coordinates into pixel (sample) and line is:

$$\begin{pmatrix} p \\ l \end{pmatrix} = \begin{pmatrix} K_x & K_{xy} & K_{xxy} \\ K_{yx} & K_y & K_{yxy} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} p_0 \\ l_0 \end{pmatrix}$$

where the $K_i$ are elements of the camera transformation matrix $K$ and $(p_0, l_0)$ are the coordinates of the camera center.
Partial Derivatives

The partial derivatives of the imaging observables with respect to parameters are needed for observation processing. To obtain the partials we first re-write the apparent position in camera body coordinates:

\[
\begin{align*}
S &= T_{SP}^T A \\
P &= T_{SP}^T S
\end{align*}
\]

with

\[
T_{SP}^T = R_3(\phi) R_2(90^\circ - \delta) R_3(\alpha) \\
T_{SP}^T = R_3(\Omega) R_1(-X) R_2(\Psi)
\]

If we wish to account for changes in scan platform orientation, we need

\[
\begin{bmatrix}
\frac{\partial S}{\partial \xi} \\
\frac{\partial S}{\partial \eta} \\
\frac{\partial S}{\partial \phi}
\end{bmatrix}
= \begin{bmatrix}
0 & -S_3 & S_2 \\
S_3 & 0 & -S_1 \\
-S_2 & S_1 & 0
\end{bmatrix}
\begin{bmatrix}
-\cos \delta \cos \phi & -\sin \phi & 0 \\
\cos \delta \sin \phi & -\cos \phi & 0 \\
\sin \delta & 0 & 1
\end{bmatrix}
\]

where \( S_1, S_2, S_3 \) are the components of the vector \( S \). If the apparent position of the object depends upon the parameter \( \xi \), we also have

\[
\frac{\partial S}{\partial \xi} = T_{SP}^T \frac{\partial A}{\partial \xi}
\]

Consequently,

\[
\frac{\partial P}{\partial \chi} = T_{SP}^T \frac{\partial S}{\partial \chi}
\]

where \( \chi \) represents \( \alpha, \delta, \phi, \) or \( \xi \). It follows that the partials of the uncorrected focal plane coordinates are

\[
\begin{bmatrix}
\frac{\partial x}{\partial \chi} \\
\frac{\partial y}{\partial \chi}
\end{bmatrix}
= \frac{f}{P_3}
\begin{bmatrix}
1 & 0 & -P_1/P_3 \\
0 & 1 & -P_2/P_3
\end{bmatrix}
\frac{\partial P}{\partial \chi}
\]

and the partials of the corrected coordinates are

\[
\begin{bmatrix}
\frac{\partial x'}{\partial \chi} \\
\frac{\partial y'}{\partial \chi}
\end{bmatrix}
= \begin{bmatrix}
1 + \frac{\partial \Delta x}{\partial x} & \frac{\partial \Delta x}{\partial y} \\
\frac{\partial \Delta y}{\partial x} & 1 + \frac{\partial \Delta y}{\partial y}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x}{\partial \chi} \\
\frac{\partial y}{\partial \chi}
\end{bmatrix}
\]

with the partials of the corrections being

\[
\begin{bmatrix}
\frac{\partial \Delta x}{\partial x} \\
\frac{\partial \Delta x}{\partial y} \\
\frac{\partial \Delta y}{\partial x} \\
\frac{\partial \Delta y}{\partial y}
\end{bmatrix}
= \begin{bmatrix}
-xy/r & r^2 + 2x^2 & -3xyr & r^4 + 4x^2r^2 & y & 2x \\
-r - y^2/r & 2xy & -r^3 - 3y^2r & 4xyr^2 & x & 0 \\
r + x^2/r & 2xy & r^3 + 3x^2r & 4xyr^2 & 0 & y \\
xy/r & r^2 + 2y^2 & 3xyr & r^4 + 4y^2r^2 & 2y & x
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4 \\
e_5 \\
e_6
\end{bmatrix}
\]
Applying the camera transformation yields the partials of the observables

\[
\left( \begin{array}{c} \frac{\partial p}{\partial \chi} \\ \frac{\partial l}{\partial \chi} \end{array} \right) = \left( \begin{array}{ccc} K_x & K_{xy} & K_{xyy} \\ K_{yx} & K_y & K_{yy} \end{array} \right) \left( \begin{array}{c} \frac{\partial x'}{\partial \chi} \\ \frac{\partial y'}{\partial \chi} \\ x' \frac{\partial y'}{\partial \chi} + y' \frac{\partial x'}{\partial \chi} \end{array} \right)
\]
Format for Imaging Observations

The imaging observations are contained in a Picture Sequence File (PSF) which is a text file consisting of a series of four different FORTRAN namelists:

1. The $ID namelist identifies the file.
2. The $CAM namelist contains data pertaining to the camera models for the cameras on the spacecraft.
3. The $PIC namelist contains variables that define a picture.
4. The $IM namelist contains variables that define one image in a picture. Depending on the nature of the image—a planet, a satellite, an asteroid, a comet, or a star—different variables are written to the output file.

The order of the namelists is as follows:

$ID
$CAM
$PIC for the first picture.
   $IM for the first image in the first picture.
   :
   $IM for the last image in the first picture.
   $IM with IMG='END' to denote the end of the first picture.
   :
$PIC for the last picture.
   $IM for the first image in the last picture.
   :
   $IM for the last image in the last picture.
   $IM with IMG='END' to denote the end of the last picture.
$PIC with PICNM='END' to denote the end-of-file.

The contents of the namelists are:

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Dim.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCID</td>
<td>C</td>
<td>1</td>
<td>The spacecraft ID.</td>
</tr>
<tr>
<td>PSFID</td>
<td>C</td>
<td>1</td>
<td>Identification of this PSF.</td>
</tr>
<tr>
<td>PSFTIM</td>
<td>C</td>
<td>1</td>
<td>Calendar date and time at which this PSF was generated.</td>
</tr>
<tr>
<td>PSFPRG</td>
<td>C</td>
<td>1</td>
<td>The name of the program that last modified this PSF.</td>
</tr>
<tr>
<td>PSFCOM</td>
<td>C</td>
<td>3</td>
<td>Additional optional documentation about the PSF.</td>
</tr>
<tr>
<td>EQUINOX</td>
<td>I</td>
<td>1</td>
<td>The equinox of the file, either 1950 or 2000.</td>
</tr>
<tr>
<td>NCAM</td>
<td>I</td>
<td>1</td>
<td>Number of spacecraft cameras.</td>
</tr>
</tbody>
</table>
### $\text{CAM}$ Namelist Variables

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Dim.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAMID</td>
<td>C</td>
<td>NCAM</td>
<td>Name of each camera.</td>
</tr>
<tr>
<td>FL</td>
<td>DP</td>
<td>NCAM</td>
<td>Focal length in millimeters of each camera.</td>
</tr>
<tr>
<td>PLCCTR</td>
<td>DP</td>
<td>2,NCAM</td>
<td>Center of each camera in pixel and line.</td>
</tr>
<tr>
<td>PLSIZ</td>
<td>DP</td>
<td>4,NCAM</td>
<td>Minimum pixel, maximum pixel, minimum line, and maximum line of the hardware field of view of each camera.</td>
</tr>
<tr>
<td>KMAT</td>
<td>DP</td>
<td>2,3,NCAM</td>
<td>The transformation matrix $K$ for each camera.</td>
</tr>
<tr>
<td>EM</td>
<td>DP</td>
<td>6,NCAM</td>
<td>The distortion coefficients for each camera.</td>
</tr>
<tr>
<td>OFFSET</td>
<td>DP</td>
<td>3,NCAM</td>
<td>Offset angles in elevation, cross-elevation, and twist from the scan platform to each camera body; expressed in degrees.</td>
</tr>
</tbody>
</table>

### $\text{PIC}$ Namelist Variables

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Dim.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PICNM</td>
<td>C</td>
<td>1</td>
<td>Name (often the FDS count) of this picture.</td>
</tr>
<tr>
<td>PICNO</td>
<td>I</td>
<td>1</td>
<td>Number of this picture on the file.</td>
</tr>
<tr>
<td>TOB</td>
<td>C</td>
<td>1</td>
<td>Calendar date of the end of the exposure in UTC.</td>
</tr>
<tr>
<td>CAMERA</td>
<td>C</td>
<td>1</td>
<td>Camera ID for this picture.</td>
</tr>
<tr>
<td>EXPTIM</td>
<td>DP</td>
<td>1</td>
<td>Exposure time, in seconds.</td>
</tr>
<tr>
<td>PICDEL</td>
<td>I</td>
<td>1</td>
<td>Picture deletion flag: 0 to keep the picture, any positive integer to delete it from processing.</td>
</tr>
<tr>
<td>RA</td>
<td>DP</td>
<td>1</td>
<td>Right ascension,</td>
</tr>
<tr>
<td>DEC</td>
<td>DP</td>
<td>1</td>
<td>declination, and</td>
</tr>
<tr>
<td>TWIST</td>
<td>DP</td>
<td>1</td>
<td>twist pointing angles of the scan platform, in degrees, with respect to inertial coordinates.</td>
</tr>
</tbody>
</table>
$IMG$ Namelist Variables

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Dim.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMG</td>
<td>C</td>
<td>1</td>
<td>Name of the image. This is the body name if the image is the center of a body or the catalog number if the image is a star.</td>
</tr>
<tr>
<td>IMGTY</td>
<td>C</td>
<td>1</td>
<td>Type of the image: ‘PLAN’, ‘SAT’, ‘ROCK’, ‘AST’, ‘COM’, or ‘STAR’ for planet centers, satellite centers, rock centers, asteroid centers, comet centers and stars, respectively.</td>
</tr>
<tr>
<td>IMGID</td>
<td>I</td>
<td>1</td>
<td>ID number of the image. This is the body code number for planets, satellites, rocks, asteroids, and comets; or the catalog number for a star.</td>
</tr>
<tr>
<td>USE</td>
<td>I</td>
<td>1</td>
<td>Image deletion flag: 0 to keep the image, any positive integer to delete it.</td>
</tr>
<tr>
<td>Z</td>
<td>DP</td>
<td>2</td>
<td>The observed pixel and line location of the image.</td>
</tr>
<tr>
<td>ZC</td>
<td>DP</td>
<td>2</td>
<td>The “local correction” which, when subtracted from Z, produces the effective pixel and line location of the image.</td>
</tr>
<tr>
<td>SIG</td>
<td>DP</td>
<td>2</td>
<td>The uncertainty in Z, in pixel and line.</td>
</tr>
<tr>
<td>STRA</td>
<td>DP</td>
<td>1</td>
<td>Right ascension and</td>
</tr>
<tr>
<td>STDEC</td>
<td>DP</td>
<td>1</td>
<td>declination of the star in degrees with respect to inertial coordinates corrected for parallax and proper motion.</td>
</tr>
</tbody>
</table>

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References


